TYPE 653: SIMPLE FLOOR HEATING SYSTEM

General Description

This component models a simple radiant slab (floor heating or cooling) system that operates under the assumption that the slab can be treated as a single lump of isothermal mass and that the fluid to slab energy transfer can be modeled using a heat exchanger effectiveness approach.

Nomenclature

$C_{\min}$	[kJ/kg]	The minimum of the slab capacitance and the fluid capacitance.
Cap_{slab}	[kJ/kg]	The capacitance of the slab.
m_{slab}	[kg/hr]	The mass of the slab.
Cp_{slab}	[kJ/kg.K]	The specific heat of the slab material.
$\dot{m}_{fluid}$	[kg/hr]	The mass flow rate of fluid passing through the slab.
Cp_{fluid}	[kJ/kg.K]	The specific heat of fluid passing through the slab.
ε	[0..1]	The effectiveness of the fluid / slab heat exchanger.
UA_{top}	[kJ/hr.K]	The heat transfer coefficient between the slab and the zone.
UA_{back}	[kJ/hr.K]	The heat transfer coefficient between the slab and the sink temperature for losses not to the zone.
$T_{fluid,in}$	[ºC]	The temperature at which fluid enters the slab
$T_{fluid,out}$	[ºC]	The temperature at which fluid exits the slab.
T_{top}	[ºC]	The temperature of the zone
T_{back}	[ºC]	The temperature to which losses from the slab occur
$\dot{Q}_{in}$	[kJ/hr]	Energy transferred from the fluid to the slab
$\dot{Q}_{zone}$	[kJ/hr]	Energy transferred from the slab to the zone
$\dot{Q}_{loss}$	[kJ/hr]	Energy transferred from the slab to the sink.

Detailed Description

Type653 operates under two main assumptions. First, that the slab can be assumed to be a lumped capacitance. Second, that the energy transfer from the fluid to the slab can be modeled using the heat exchanger effectiveness method. Combined, these assumptions mean that the radiant slab can be modeled using a simple differential equation of the form dT/dt = aT + b where a and b are constants.

In making a lumped capacitance assumption concerning the slab, it is assumed that there are no temperature gradients throughout the slab as it heats up and cools down: that the slab is isothermal throughout. This is obviously an idealized assumption since it is temperature gradients that drive conduction heat transfer within the slab. However, if the internal resistance to heat transfer is high in comparison to the rate at which energy is transferred away from the surface of the slab, it is reasonable to assume that the slab is isothermal and that it can be treated as a lumped capacitance.

The effectiveness approach to modeling heat exchangers is a helpful simplification of the complex and piping configuration dependent heat transfer that goes on between the two heat exchange mediums. In

the case of fluid filled pipes embedded within a slab, the two mediums between which energy is exchanged are the fluid and the slab material. The effectiveness of a the heat exchanger is defined as the actual energy transferred divided by the maximum possible energy transfer between the two mediums. In the slab, the maximum possible energy transfer would occur if the fluid exited the slab at the slab temperature or if the slab temperature rose to the fluid inlet temperature. The medium (slab or fluid) which could undergo the maximum energy transfer would be the one with the minimum capacitance because the energy balance requires that the energy given up by one medium must be absorbed by the other. If the medium with the larger capacitance undergoes the maximum temperature difference, this would cause the other medium to go through a temperature change larger than the maximum in order for the energy balance to work out. The minimum capacitance side of the fluid / slab heat exchanger can be found using equation 653.1.

$$C_{\min} = MIN\left(\left(\dot{m}_{fluid}\ Cp_{fluid}\right),\left(m_{slab}\ Cp_{slab}\right)\right) \qquad \text{(Eq. 653.1)}$$

The slab mass multiplied by the slab specific heat is refered to as the slab capacity, a PARAMETER of the Type653 model. Figure 1 shows a schematic of the energy balance on the slab / fluid system.

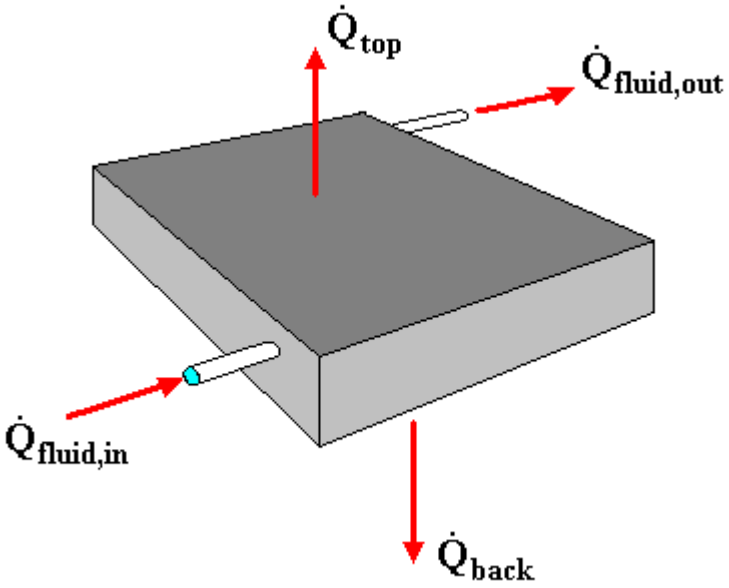


Figure 1: Slab / Fluid System Energy Balance

The energy balance can be written mathematically as:

$$m_{slab}\ Cp_{slab}\ \frac{dT_{slab}}{dt} = -UA_{top}\left(T_{slab} - T_{top}\right) - UA_{back}\left(T_{slab} - T_{back}\right) + \varepsilon\ C_{\min}\left(T_{fluid,in} - T_{slab}\right) \qquad \text{(Eq. 653.2)}$$

TRNSYS contains two methods for solving differential equations. If the differential equation can be written in the form dT/dt = aT + b, (in which both a and b are constants) then the equation can be solved analytically by calling a Differential Equation solving subroutine. If the equation cannot be written in that form, then TRNSYS can rely upon its ability to iterate at a given time step until all connected outputs have converged in order to solve the differential equation. In the case of the lumped capacitance

slab, the energy balance can be written in the form dT/dt = aT + b. The "a" and "b" terms are given in equations 653.3a and 653.3b respectively.

$$a = \frac{-\varepsilon\, C_{\min} - UA_{zone} - UA_{loss}}{m_{slab}\, Cp_{slab}} \qquad \text{(Eq. 653.3a)}$$

$$b = \varepsilon\, C_{\min} \frac{T_{fluid,in}}{m_{slab}\, Cp_{slab}} + UA_{top} \frac{T_{top}}{m_{slab}\, Cp_{slab}} + UA_{back} \frac{T_{back}}{m_{slab}\, Cp_{slab}} \qquad \text{(Eq. 653.3b)}$$

During any given iteration, Type653 passes the temperature of the slab at the beginning of the time step to the TRNSYS Differential Equation solving subroutine. This routine returns the temperature of the slab at the end of the time step. With the slab temperature at the end of the time step, Type653 then calculates three energy transfers. The energy transferred to the slab from the fluid stream is:

$$\dot{Q}_{in} = Cap_{\min}\left(T_{fluid,in} - T_{slab}\right) \qquad \text{(Eq. 653.4)}$$

The energy transferred from the slab to the zone is:

$$\dot{Q}_{zone} = UA_{top}\left(T_{slab} - T_{top}\right) \qquad \text{(Eq. 653.5)}$$

And the energy transferred from the slab to the sink temperature is:

$$\dot{Q}_{loss} = UA_{back}\left(T_{slab} - T_{back}\right) \qquad \text{(Eq. 653.6)}$$

The temperature of fluid exiting the slab is given by

$$T_{fluid,out} = T_{fluid,in} - \frac{\dot{Q}_{in}}{\dot{m}_{fluid}\, Cp_{fluid}} \qquad \text{(Eq. 653.7)}$$